

## **BADRI NATH PRASAD**

**(1899-1966)**

**Elected Fellow 1936**

### **CHILDHOOD AND FORMATIVE INFLUENCES**

BORN on January 12, 1899, of a well-known respectable family of businessmen of Mohammadabad-Gohna in the Azamgarh District of the State of Uttar Pradesh in India, Badri Nath Prasad was the youngest son of his parents. A man of foresight and practical wisdom, his father, Sri Ram Lal, succeeded in giving the best possible education to his sons. Full of promise in his boyhood, young Prasad had a remarkably strong memory. He used to recall, how in his school days he could perform the amazing feat of memorizing even long verses by reading them only once or twice. In shaping his personality, his mother and elder brother seem to have exercised an influence even greater than his father did. With a feeling of reverence, he used to recall how on every occasion he paid his obeisance to his mother, who blessed him, "Go, Be Victorious."

### **SCHOOL AND UNIVERSITY EDUCATION**

Young Prasad started his early education by learning the Urdu language, but later on he switched over to Hindi, thereby obtaining an almost equal command over both the languages. Early in his school career he studied in his home town Mohammadabad-Gohna and the C. A. V. High School of Allahabad (with his elder brother who was then studying in the Allahabad University), and passed the Matriculation Examination of the Calcutta University from the V. M. H. E. School, Siwan (Bihar), and the I. Sc. and B. Sc. examinations of the same University from the Patna College, Patna, with a uniformly brilliant record. During these years he was particularly fond of playing hockey. For his M. Sc. degree in Mathematics, Prasad studied at the Banaras Hindu University and obtained a first class first both in the Previous and Final year examinations in 1920 and 1921 respectively securing a higher percentage of marks than any other candidate.

### **INFLUENCE OF DR GANESH PRASAD**

While studying for his M. Sc. degree, B. N. Prasad came in close contact with his teacher Dr Ganesh Prasad, who, recognizing his talents, inspired him to take to teaching and creative research in Pure Mathematics. In those days, desire to join the Civil Services was almost a craze among bright Indian students. On the basis





B.N. Prasad



of his brilliant result in the M. Sc. examination, Prasad was actually offered a lucrative assignment. Members of his family urged him to accept this offer, but he declined it and out of love for Mathematics and reverence for his teacher and followed Dr Ganesh Prasad's advice. Such was the loving influence of this great teacher on his pupils. So, in July 1921, B. N. Prasad got himself enrolled as a research scholar in the Department of Mathematics of the Banaras Hindu University to work under the supervision of Dr Ganesh Prasad. His appointment was that of an Assistant Professor of Mathematics and this made him shoulder a heavy teaching load, but Prasad's zeal and interest in advanced topics were so great that he published two original papers on the properties of non-differentiable functions. Dr Ganesh Prasad moved to the Calcutta University in 1923, and B. N. Prasad joined the Allahabad University on July 17, 1924 as a Lecturer in Mathematics. A number of his good students migrated with him to Allahabad.

### RESEARCH CAREER ABROAD

As a young lecturer B. N. Prasad had to shoulder a heavy teaching load at Allahabad also and this retarded the progress of his research work to some extent. But during the first five years he spent at Allahabad, he devoted himself to an intensive study of the work of eminent mathematicians of Europe and England, like W. H. Young, G. H. Hardy, J. E. Littlewood, E. C. Titchmarsh, H. Lebesgue, A. Denjoy and others on the theory of trigonometric series. Their work fascinated him so much that B. N. Prasad obtained study leave from the Allahabad University and proceeded in 1929 to the United Kingdom. Initially, he stayed for a few months at Edinburgh and did some work on the theory of Fourier series, but thereafter went to Liverpool to work with Professor E. C. Titchmarsh. He completed his thesis for the Ph. D. degree of the University of Liverpool in 1931. Highly impressed by his work, Professor Titchmarsh, writing about Prasad, said : "*I found him an extremely industrious and intelligent worker who had plenty of ideas of his own and merely asked for my advice and direction.*"

After completing his Ph. D. thesis at Liverpool, B. N. Prasad went to Paris to work with the famous French mathematician Arnaud Denjoy. Under the guidance of Professor Denjoy, Prasad prepared in less than a year his well-known thesis entitled : *Contribution a l' étude de la séries conjuguée d'une série de Fourier*. This thesis earned him the degree of *Docteur ès Science* of the University of Paris with mention *très honorable*. The 'soutenance' of his thesis for the *docteurat d'Etat* took place on June 4, 1932, before a Jury consisting of Professor Emile Borel as President and Professor A. Denjoy and Professor G. Valiron as examiners. The quality of his thesis greatly impressed the examine and Professor Borel, while announcing the result, paid compliment to Prasad, saying (translation):

*"You have shown particular merit in writing your thesis in French language and in showing proof, in its defence, of a very deep knowledge of our language. We know that a magnificent development of science, and articularly of mathematical science, has taken place in your great and beautiful country. We can only rejoice to see one of the most distinguished among young Indian mathematicians associating*



himself through his work with the French Mathematical School. Science can make no progress unless by the conjugate efforts of all the nations, for science is essentially international. This latter national character shows itself by the liasion which exists in the work of mathematicians of different countries and by their reciprocal influences on each other. Thanks to you, one of these precious contacts finds itself established between your country and ours. We cordially wish that the relations which have begun in this way, will develop to the common advantage of both our countries and of science."

B. N. Prasad thus became the first Indian to receive the State D. Sc. degree in Mathematics from the University of Paris, a creditable achievement indeed! His teacher Professor Denjoy was so impressed by Prasad's work that in 1961 in his message of good wishes at the time of Prasad's retirement from active service of Allahabad University, he wrote : (translation) :

*"About 35 years ago, Professor B. N. Prasad came to Paris to complete the work of his thesis for the degree of Doctor of Science of State-France. He was kind enough to associate myself with the progress of his research work. I could admire week to week, if not day to day, the command with which he handled the more subtle and the more delicate instruments of mathematical analysis for putting in evidence of the propriety of the trigonometric series, independent or conjugate. I remember that in examining his demonstrations, his transformations of the calculus, I imagined a watchmaker, who has to select tools more and more fine, more and more precise, for separating minute wheels.*

*Returning to India soon after defending his thesis—and in a very brilliant manner—Mr B. N. Prasad had to render in his country eminent services to Indian Mathematics."*

Taking advantage of his stay abroad, B. N. Prasad visited centres of mathematical research in England, Scotland, Ireland, France, Germany and Italy and came in personal contact with celebrated mathematicians like Hadamard, Hilbert, Whittaker, Hardy, Lebesgue and others. He returned to India in July 1932 and resumed his duties as Lecturer at the Allahabad University.

## PROFESSIONAL CAREER

Although Dr Prasad returned to Allahabad with conspicuous academic achievements, he failed to get recognition from his employers, the Allahabad University. For full 14 years (upto 1946) he remained a Lecturer, and for another 14 years (till August 1960) he remained a Reader and was appointed Professor only towards the end of his career. It was sheer irony that a person whom any university in India would have been proud to have as its Mathematics Professor 20 years earlier, he chose to serve the Allahabad University as a Lecturer and Reader practically for the entire period of his most productive career.

But these difficulties in the university, could hardly subdue the spirits of Dr Prasad. His keen desire to create an active research centre in Analysis at Allahabad remained throughout uppermost in his mind. He encouraged and inspired bright young students to pursue higher studies and take to a career of teaching and research. Those were the days when there were few scholarships. Salary scales in the



universities were low and employment opportunities were few and above all, the brighter students preferred the administrative services. Undaunted by these handicaps, Dr Prasad succeeded, during 1932–40, in training a band of devoted research workers. Through his indomitable courage and determination, he kept the torch of mathematical research burning at Allahabad. It is gratifying that ultimately his efforts bore fruit; during the forties and fifties, a good number of bright young researchers gathered round him making Allahabad a leading centre of research in Analysis. He succeeded in building at the Allahabad University an active School of Research on Summability Theory, which developed and flourished during his life time and which is appropriately associated with his name. Four of his research pupils were awarded the D. Sc. degree and over a dozen the D. Phil. degree by the Allahabad University, and these have kept research in Summability Theory alive both at Allahabad and at other centres in India. Under his supervision over 150 research papers were prepared and published in various national and international mathematical research journals of repute. When the Government of India instituted National Research Fellowships in 1956, two of them in Mathematics were awarded to Prasad's pupils on the basis of their research work. These were in addition to many others awarded in subsequent years.

Dr B. N. Prasad's teaching career spread over nearly 40 years, out of which  $34\frac{1}{2}$  years were spent at Allahabad which he did not choose to leave, although many a time he received offers for higher posts from several universities in India. However, once, in March 1949, he yielded to pressure and accepted an invitation from the Government of Bihar to join the Science College, Patna, as Professor and Head of the Department. But realizing soon that the atmosphere there was not congenial to his academic pursuits, he resigned his professorship and came back to Allahabad in January 1951 to his old post.

Dr Prasad was primarily interested in research, but he was loved and admired by his under-graduate and post-graduate students, for he was an excellent teacher and a source of inspiration to generations of young students. He was a real *guru* in the traditional Indian sense. He helped scores of young men and women (including the author of this memoir) to chalk out suitable and dignified careers for themselves. But he always remained a hard taskmaster and that was perhaps the secret of his success as a research guide.

With the object of creating among students an interest in Mathematics and encouraging original work in the subject, on August 28, 1924, B. N. Prasad founded in the Allahabad University a Mathematical Association with Professor A. C. Banerji, the then Head of the Mathematics Department as President and himself as Secretary. The Association published an Annual Bulletin containing articles written by students and teachers of the Mathematics Department.

During his brief period as Head of the Department (November 1, 1958–January 11, 1961), several noteworthy events took place in the Mathematics Department. For the first time, in December 1958, a Mathematics Exhibition was organized at the Allahabad University which drew praise from distinguished visitors like Sri Jawaharlal Nehru, Sir K. S. Krishnan, Dr H. J. Bhabha and others. Again for the first time during its 50-year existence, in 1959, the Indian Mathematical Society was invited by Dr Prasad to hold its 25th Conference at the Allahabad University.



On the basis of Building Grants sanctioned by the University Grants Commission, through his efforts, Dr Prasad got the Foundation-Stone of the much-needed Mathematics Department Extension Building laid on March 8, 1960. During his regime two important special papers on Numerical Analysis and Linear Operators were introduced for the students of the M. A./M. Sc. (Final) classes. Again, it was through his efforts that the Library of the Department was strengthened by the acquisition of books and journals. When Dr Prasad retired from the Allahabad University on January 11, 1961, a Special Commemorative Issue of the Bulletin of the Mathematical Association was brought out and a memorable and touching farewell given to him by his pupils and friends.

### CONTRIBUTIONS TO MATHEMATICS

The name B. N. Prasad has come to be associated with an extensive cross-section of Modern Analysis including diverse subjects like the Theory of Fourier Series, its conjugate series, their derived series, the theory and applications of absolute summability of infinite series in general, summability factors, Fourier integrals, the concept of radial variation, non-summability in the sense of Abel, the theory of generalized derivatives, strong summability—especially with Riesz' typical means, the Gibbs phenomenon and the generalized jump of a function, the multiplication of Dirichlet series and the second theorem of consistency.

The many D. Sc. and D. Phil. theses submitted from time to time under his supervision and the research papers published round the world in the wake of his own pioneering contributions on these subjects, bear ample testimony to the intrinsic mathematical significance and power of his outstanding work.

In order to give a brief resumé of the important researches of Dr Prasad, it will be convenient to introduce at the outset certain definitions and notations.

Let  $f(x)$  be a periodic function, with period  $2\pi$ , integrable in the sense of Lebesgue over  $(-\pi, \pi)$ , and let its Fourier series be taken (without any loss of generality) to be given by

$$f(x) = \frac{1}{2}a_0 + \sum_{n=1}^{\infty} (a_n \cos nx + b_n \sin nx) \equiv \sum_{n=1}^{\infty} A_n(x). \quad \dots(1)$$

Then the series

$$\sum_{n=1}^{\infty} (b_n \cos nx - a_n \sin nx) \equiv \sum_{n=1}^{\infty} B_n(x), \quad \dots(2)$$

is called the conjugate series of the Fourier series of  $f(x)$ . We shall also write

$$\left. \begin{aligned} \phi(t) &= \frac{1}{2} [f(x+t) + f(x-t) - 2f(x)], \\ \psi(t) &= f(x+t) - f(x-t) \\ \Phi(t) &= \int_0^t \phi(u) du. \end{aligned} \right\} \quad \dots(3)$$

Denoting by  $S_n^\alpha$  the  $n$ th Cesàro mean of order  $\alpha$  of the sequence of partial sums  $\{s_n\}$  of any given infinite series  $\sum u_n$ , we say that  $\sum u_n$  is summable  $(C, \alpha)$  to the sum  $S$ , if  $\lim_{n \rightarrow \infty} S_n^\alpha = S$ , and summable  $|C, \alpha|$  if  $\sum |S_n - S_{n-1}| < \infty$ . If  $\sum u_n x^n$  is convergent for  $|x| < 1$  and equal to  $f(x)$ , then  $\sum u_n$  is said to be summable  $(A)$  to  $S$ , if  $\lim_{n \rightarrow \infty} S_n = S$ , and summable  $|A|$  if  $f(x) \in BV(0, 1)$ . If we write



$$R_{\lambda}^k(\omega) = \omega^{-k} \sum_{\lambda_n < \omega} (\omega - \lambda_n)^k u_n, \quad k \geq 0,$$

then  $\sum u_n$  is said to be summable  $(R, \lambda_n, k)$  to  $S$ , if

$$\lim_{\omega \rightarrow \infty} R_{\lambda}^k(\omega) = S,$$

and summable  $|R, \lambda_n, k|$  if  $R_{\lambda}^k(\omega) \in BV(A, \infty)$ , for some finite  $A > 0$ .

### Convergence and Summability of Fourier Series

A generalization of the classical result of Fejér, the well-known result of Lebesgue that the Fourier series of a periodic function which is integrable ( $L$ ) is summable  $(C, 1)$  almost everywhere, was indeed one of the turning points in the theory of summability of Fourier series. Hardy proved in 1913 that the order of summability could be reduced to  $\delta > 0$ . The set of points considered by Lebesgue was the set of points  $x$ , at which  $\int_0^t |\phi(u)| du = o(t)$ , as  $t \rightarrow \infty$ , the so-called 'Lebesgue set' which certainly is more general than the set of points at which  $\phi(t) = o(1)$ , as  $t \rightarrow 0$ . Indeed, Lebesgue's criterion may be valid even at the points of discontinuity of the second kind. The fundamental result of Lebesgue was generalized in another direction by Noaillon and Young and ultimately by Hardy by replacing the condition that  $x$  belongs to the Lebesgue set by the couple of conditions

- (i)  $(1/t) \int_0^t \phi(u) du$  exists as  $t \rightarrow 0$ , and
- (ii)  $\int_0^t |\phi(u)| du = O(t)$ ,

which may be satisfied even at points outside the Lebesgue set, but which are of course satisfied at points of the Lebesgue set. Cases were, however, known in which the Fourier series was summable  $(C, 1)$  even when the limit of  $\Phi(t)/t$ , as  $t \rightarrow 0$ , does not exist. One of the problems that Prasad successfully tackled (1934) was the question of the Cesàro summability of the Fourier series of  $f(t)$ , at a point  $t = x$ , where this limit fails to exist. The basic technique developed by Prasad for the solution of this problem in the case of  $(C, 1)$  summability is to reduce the discussion of the  $(C, 1)$  summability of  $f(t)$ , at  $t = x$ , that is, of  $\phi(t)$ , at  $t = 0$ , to the convergence of the Fourier series of  $\Phi(t)/t$ , i.e.,  $\phi_1(t)$ , at the same point. In thus reducing the problem of summability to an associated problem of convergence, Prasad had automatically been led to a number of criteria of summability  $(C, 1)$  depending upon the already known criteria for convergence of a Fourier series at a point. He proved, *inter alia*, that the existence of the integral  $\int_0^{\delta} \Phi(t) t^{-2} dt$  is sufficient for the summability  $(C, 1)$  of the Fourier series of  $f(t)$ , at  $t = x$ . This criterion may be applicable even when the limit of  $\Phi(t)/t$  does not exist, as is demonstrated by the following example :

$$\Phi(t) = n^{-2} \sin^2 \{n^4 (t - n^{-2})\pi\} \quad (n^{-2} \leq t \leq n^{-2} + n^{-4}),$$

and  $\Phi(t) = 0$ , elsewhere.

It is known that the following three conditions are in descending order in respect of their degrees of stringency, so that each will hold if the preceding one holds, but not vice versa :

$$\int_0^{\delta} |\phi(t)/t| dt \text{ exists,} \quad \dots(4)$$

$$\int_0^{\delta} \{\Phi(t)/t\} dt \text{ exists as a non-absolutely convergent integral} \quad \dots(5)$$



and  $\Phi(t) = o(t)$ , as  $t \rightarrow 0$ . ... (6)

It is also known that when (4) holds, the Fourier series is convergent by Dini's condition and hence also summable  $(C, 1)$ . When (5) is satisfied, it is known that the Fourier series is not necessarily convergent. Hahn had investigated if, at a point  $x$ , the holding of the condition (6) is a sufficient condition for the summability  $(C, 1)$  of the Fourier series of  $f(t)$ , at  $t = x$ , and he had construed an example to prove that the answer was in the negative. The more delicate investigations as to the summability or non-summability of Fourier series, when the more stringent condition (5) is satisfied, were taken up by Prasad, who showed (in 1935) by constructing an example that even when the condition (5) is satisfied, the Fourier series, although summable  $(C, 1 + \delta)$  for every  $\delta > 0$ , is not necessarily summable  $(C, 1)$ .

### *Convergence and Summability of the Conjugate Series of a Fourier Series*

Prasad's contributions to the theory of convergence and summability  $(C)$  and  $(A)$  of the conjugate series have had far-reaching influence on later developments. It is well known that a conjugate series is not necessarily a Fourier series. It had been customary to associate with the conjugate series the conjugate function  $g(x)$  given by

$$\left. \begin{aligned} g(x) &= (1/2\pi) \int_0^\pi \{f(x+t) - f(x-t)\} \cot \frac{1}{2}t \, dt \\ &= (1/2\pi) \int_0^\pi \psi(t) \cot \frac{1}{2}t \, dt \\ &= (1/\pi) \int_0^\infty \psi(t) / t \, dt, \end{aligned} \right\} \dots (7)$$

on account of the periodicity of  $f(t)$ , and to look upon this almost *a priori* as the sum function and then discuss the convergence and summability of the conjugate series, at the point  $t = x$ , to  $g(x)$  under suitable additional restrictions imposed upon  $\psi(t)$ . Prasad not only gave general criteria (1931) for the convergence of the conjugate series, at a point, but also made (1931-32) a rather comprehensive study of its  $(C, 1)$  summability. By means of a technique, parallel to the one he had used for the Fourier series, he deduced a number of  $(C, 1)$ -summability criteria, some of which may be mentioned as follows.

If the integral  $g(x)$  exists at least as a non-absolutely convergent integral, then the conjugate series is summable  $(C, 1)$  to  $g(x)$  provided

$$\text{the integral } \int_0^\delta |\psi(t)| t^2 \, dt \text{ exists} \dots (8)$$

$$\text{or } (1/t) \int_0^t du \cdot (1/u) \int_0^u \psi(v) \, dv \text{ is of bounded variation} \dots (9)$$

$$\text{or } \int_0^t |\psi(t)| \, dt = O(t). \dots (10)$$



The criterion (10) includes as a particular case Young's criterion for the summability  $(C, 1)$  of the conjugate series, at  $t = x$ , viz., if  $g(x)$  exists then the conjugate series is summable  $(C, 1)$  to  $g(x)$ , provided

$$\int_0^t |\psi(t)| dt = o(t).$$

But Prasad raised a crucial question : What can be asserted about the  $(C, 1)$  summability at a point  $x$ , at which  $g(x)$  fails to exist ? Prasad proved that the earlier theorems on  $(C, 1)$  summability including those corresponding to the criteria (8), (9) and (10), continue to be true even when  $g(x)$  fails to exist, provided, what Prasad terms, the 'generalized conjugate function'

$$G(x) = (1/4\pi) \int_0^\pi \Psi(t) \cos^2 \frac{1}{2}t dt \quad \dots(11)$$

exists. Naturally, this result had a far-reaching effect on the theory of Cesàro summability of the conjugate series.

It was known that the integral  $g(x)$  exists for almost all values of  $x$  and also that the summability  $(C, \delta)$ , for  $\delta > 0$ , of the conjugate series for almost every  $x$ , more precisely for every  $x$  where

$$\int_0^t |\psi(u)| du = o(t),$$

as  $t \rightarrow 0$ , is equivalent to the existence of the function  $g(x)$  so that it finally emerges that the conjugate series is summable  $(C, \delta)$  almost everywhere. This too is readily deducible from the well-known theorem of Hardy and Littlewood that in the Lebesgue set the conjugate series is either summable by every Cesàro mean of positive order or summable by no Cesàro mean in conjunction with the fact that the conjugate series is summable  $(C, 1)$  at every point of the Lebesgue set. Thus for almost all points a necessary and sufficient condition for the summability  $(C, \delta)$ ,  $\delta > 0$ , of the conjugate series is the existence of  $g(x)$ . Prasad had shown that if  $f(t)$  is bounded then the conjugate series is summable  $(C, \delta)$ , for every  $\delta > 0$ , to  $g(x)$ , whenever  $g(x)$  exists. Also Hardy and Littlewood showed that for such an  $f(t)$  the existence of  $g(x)$  is a necessary condition for summability  $(C, \delta)$ , for  $\delta > 0$ , of the conjugate series at  $t = x$ .

The question naturally arose whether the existence of  $g(x)$  by itself is sufficient to ensure the summability  $(C, \delta)$ ,  $\delta > 0$ , of the conjugate series at  $t = x$ . Prasad proved (1932) that this is not so in general, even for  $\delta = 1$ , by constructing a negative example. The construction of such an example, however, essentially depends upon a suitable type of another example, for which, at a point, the original function is continuous and also the integral  $g(x)$  exists, but still the corresponding conjugate series fails to converge at that point. In case the conjugate function  $g(x)$  does not exist, but the generalized conjugate function  $G(x)$  exists, Prasad has proved that the conjugate series will still be summable  $(C, \delta)$ , for  $\delta > 1$ , provided

$$\Psi(t) = o(t), \text{ or even } \int_0^t |\Psi(t)/t| dt = o(t).$$



### Summability $(C, r)$ , $r > 0$ , of the Conjugate Series

Prasad developed a merit by which sufficient conditions for summability  $(C, r)$ ,  $r > 0$ , of the conjugate series as also the sum to which it is summable could be obtained in a simple manner. He proved that if

$$\chi_r(t) = \int_0^t (dt_1/\sin t_1) \int_0^{t_1} (dt_2/\sin t_2) \dots \int_0^{t_{r-2}} (dt_{r-1}/\sin t_{r-1}) \int_0^{t_{r-1}} \Psi(t_r) dt_r'$$

$$\chi_1(t) = \int_0^t \psi(u) du = \omega_1(t),$$

$$\omega_r(t) = \int_0^t (dt_1/t_1) \int_0^{t_1} (dt_2/t_2) \dots \int_0^{t_{r-2}} (dt_{r-1}/t_{r-1}) \int_0^{t_{r-1}} \psi(t_r) dt_r'$$

and if the  $(r + 1)$  the generalized integral

$$G_{r+1}(x) = (1/4\pi) \int_0^\pi \chi_{r+1}(t) \operatorname{cosec}^2 \frac{1}{2}t dt$$

exists, then the conjugate series is summable  $(C, r)$  to  $G_{r+1}(x)$ , provided that

$$\lim_{n \rightarrow \infty} \int_0^\delta [(\omega_r(t))/t] (\cos nt)/t dt = 0, 0 < \delta < \pi.$$

From this he obtained tests for the summability  $(C, r)$  of the conjugate series, corresponding to those for the summability  $(C, 1)$ .

The results of Prasad have since been extended by some of his students and also by other mathematicians but the merits of his work lies in the introduction of the generalization  $G(x)$  of the conjugate function  $g(x)$ . This idea of Prasad opened new avenues for further research. His methods are simple and elegant, his tools mostly classical, but he used them with great skill to arrive at important results of profound significance.

### Abel Summability of the Conjugate Series

A method of summability which is more delicate than the Cesàro summability of any positive order is that due to Poisson—now more familiar as the Abel summability method. Applying this method to the study of the conjugate series at a point, Fatou had proved (1906) that if

$$V(r, x) = \sum_{n=1}^{\infty} (b_n \cos nx - a_n \sin nx) r^n, 0 \leq r < 1, \epsilon = \sin^{-1}(1-r),$$

then

$$\lim_{r \rightarrow 1} [V(r, x) - (1/2\pi) \int_\epsilon^\pi \psi(t) \cot \frac{1}{2}t dt] = 0,$$

provided that  $f(t)$  is a bounded and continuous function. Lichtenstein showed that for the truth of this result it is sufficient that  $x$  be a point of continuity of  $f(t)$ . Later Plessner (1923) and Prasad (1931) demonstrated that the result holds for every point  $t = x$ , for which  $\Psi(t) = o(t)$  as  $t \rightarrow 0$ . Subsequently, Prasad showed



that the result would remain true in a more general sense under a less stringent condition; more precisely, he showed that

$$\lim_{r \rightarrow 1} [V(r, x) - (1/4\pi) \int_{\epsilon}^{\pi} \Psi(t) \operatorname{cosec}^2 \frac{1}{2}t dt] = 0,$$

provided that

$$\lim_{t \rightarrow 0} (1/t) \int_0^t [\Psi(t)/t] dt = 0.$$

Prasad also showed (1932) that if the integral

$$g(x) = (1/2\pi) \int_0^{\pi} [f(x+t) - f(x-t)] \cot \frac{1}{2}t dt$$

diverges to  $+\infty$  or  $-\infty$ , the Abel-limit of the conjugate series at  $t = x$  will also diverge to the same value.

### *Absolute Summability : | A | method*

The concept of summability, as a generalization of convergence, had become well known even before the beginning of the 20th century, but not until 1911 the concept of absolute summability as the corresponding generalization of absolute convergence was developed. While the initial work on absolute Cesàro summability was done by Fekete and Kogbetliantz by 1925, the absolute Riesz summability was brought into existence in 1929 by Obrechhoff. It was a coincidence that when in early thirties Prasad began his work in England on absolute summability, Whittaker defined the method of absolute Abel summability. He also showed that Dini's convergence criterion was also a criterion for the summability | A | of a Fourier series at a point. Taking up the thread immediately, Prasad brought about a striking improvement in the then existing situation by proving that Jordan's convergence criterion was also a | A | summability criterion for the Fourier series at a point. He also established a number of results with increasing degrees of generality on the absolute Abel summability of the conjugate series at a point. There is, however, one particular contribution by Prasad, which is of a very specific nature. Whittaker had shown it to be possible for a Fourier series to converge at a point without being summable—| A | at that point. The natural question which arose thereafter was whether absolute Abel summability implied convergence at a point. Prasad settled this question in 1930 by demonstrating that a Fourier series may be summable | A | at a point without being convergent there, thus establishing the independence of convergence and summability | A | of Fourier series.

Although Prasad's work on the absolute Abel summability of Fourier series and its conjugate series greatly influenced the work of contemporary mathematicians like Bosanquet, Kuniyeda, M. L. Misra, Takahashi, Hyslop and others, significant contributions made by Bosanquet need a special mention, which emerged from his attempt at generalizing Prasad's result on absolute Abel summability and were concerned with the more stringent absolute Cesàro summability of a



Fourier series and its conjugate series at a point. It will be no exaggeration to state that it were the researches of the three pioneers—Whittaker, Prasad and Bosanquet—which gave the initial impetus for most of the work on the absolute summability of the Lebesgue-Fourier series.

### *Absolute Summability Factors*

One of the most interesting aspects of the recent researches on Fourier series pertains to the determination of suitable factors  $\{\lambda_n\}$  such that the series  $\Sigma \lambda_n A_n(t)$  may be summable in some prescribed sense. Such factors are known as 'summability factors' of Fourier series. When the summability method in question is absolute, these factors are known as 'absolute summability factors' of Fourier series.

The first applications of absolute summability factors of Fourier series were due to Prasad (1933). He proved the following theorem :

If  $\{\lambda_n\}$  be any of the sequences

$$[\log n]^{1+\epsilon}]^{-1}, [\log n (\log \log n)^{1+\epsilon}]^{-1}, \dots, [\log n (\log \log n) \dots \dots (\log \dots \log_p n)^{1+\epsilon}]^{-1}, \epsilon > 0, \quad \dots(12)$$

then  $\Sigma \lambda_n A_n(t)$  is summable  $|A|$  at every point  $t = x$ , where

$$\int_0^t |\phi(u)| du = o(t), \quad \dots(13)$$

as  $t \rightarrow 0$ .

Since the condition (13) holds for almost all values of  $x$ , it follows that factors of the type (12) are  $|A|$ -summability factors of the Fourier series  $\Sigma A_n(t)$  for almost all values of  $t$ . This result has its analogues for the conjugate series and restricted Fourier series of second class, which also Prasad established in course of his researches. This result of Prasad formed the basis of a series of results proved in later years by Izumi and Kawata (1938), Cheng (1948), Pati (1954), Pati and Sinha (1958) and others.

Using his results relating to summability  $|A|$  of trigonometric series, Prasad investigated conditions under which the sum of a power series, with the unit circle as its circle of convergence, is a function of bounded variation on a radius vector, and proved important results in this regard. The following is one of them :

If a power series  $\Sigma c_n z^n$  ( $z = r e^{i\theta}$ ) with bounded coefficients is such that its second integrated series, which necessarily converges uniformly on the circle of convergence, has for sum a function of  $\theta$ , whose second order derivative is a function of bounded variation, for all points of an arc, then the sum of the power series is a function of bounded variation on every radius vector in that arc.

### *Work done in Collaboration with Pupils*

Izumi and Kawata and Cheng extended Prasad's result on  $|A|$ -summability factors of Fourier series while Pati obtained generalizations which contained both of these results as special cases. Prasad and Bhatt gave further extensions (1957) and a number of allied results of various kinds in the same direction.

Prasad and Siddiqi extended the scope of summability of the derived Fourier series (1949) by applying Nörlund means instead of Cesàro means, which had been used earlier by Chen and others. Prasad and Siddiqi later obtained a very



general result concerning the Nörlund summability of the  $r$ -th derived Fourier series, generalizing earlier results proved by Astrachan, Zygmund and Wang. This result appeared in 1950. In 1958 they obtained very detailed results on the determination of the *jump of a function* by means of the Nörlund means of its Fourier series and on the *Gibbs phenomenon* studied in similar context.

Prasad and U. N. Singh worked on the strong summability of order unity of the derived series of a Fourier series at a point, the generating function being assumed to be of bounded variation in the fundamental interval. This result was a source of inspiration for some of the subsequent researches on the subject.

To Obrechhoff is due the first theorem of consistency for absolute Riesz summability which asserts that if a series is summable  $|R, \lambda, \alpha|$ ,  $\alpha \geq 0$ , then it is also summable  $|R, \lambda, \beta|$  for every  $\beta > \alpha$ . The consequence of this theorem is that if the type remains unchanged, the effectiveness of the absolute Riesz summability method increases with the order. Since summability  $|R, \lambda, 0|$  is equivalent to absolute convergence, it is obvious that the Riesz summability method is an absolutely regular one.

The inclusion relation of the kind  $|R, \lambda_n, k| \subset |R, \phi(\lambda_n), k|$ , where the types are different but the orders are identical, constitute what is called the 'second theorem of consistency' for the absolute Riesz summability. The general result in this direction is that if the order of summability remains the same, then for the absolute summability, the efficiency of the Riesz means increases, or at any rate does not decrease, as the rate of increase of the type decreases. The first theorem in this direction was obtained by Chandrasekharan (1942) as analogue for absolute summability of Hardy's well-known extension of the classical 'Second Theorem of Consistency' for ordinary Riesz summability. This theorem was generalized (1954) by Pati for integral orders. An improvement on Pati's result was effected by Guha (1956), who also established a result for the case of non-integral order of summability. For non-integral orders, Prasad (1958) and Prasad and Pati (1957, 1960) obtained results in which great simplicity has been effected on the conditions required. They have proved :

If  $\phi(t)$  is a non-negative, monotonic increasing function of  $t$ , for  $t \geq 0$ , steadily tending to infinity as  $t \rightarrow \infty$ , such that  $\phi(t)$  is a  $(\kappa + 1)$ th indefinite integral, for  $t \geq 0$ ,  $\kappa$  being the integral part of  $k$  (assumed non-integral),

$$t^{k+1} \phi^{(k+1)}(t) / \phi(t) \in B(h, \infty),$$

where  $h$  is a finite positive number, and, uniformly in  $0 < v < 1$  and  $s > 0$ ,

$$\Lambda = s^\kappa \left\{ \frac{(1-v)t}{\phi(s+t) - \phi(s+vt)} \right\}^{k+1-\kappa} \left[ \frac{\{\phi^{(1)}(s+vt)\}^{k+1}}{\{\phi(s+t)\}^\kappa} \right] \in BV_t(h, \infty),$$

then any infinite series which is summable  $|R, \lambda_n, \kappa|$  is also summable  $|R, \phi(\lambda_n), \kappa|$ . In particular, the result of the above theorem remains true if  $\Lambda$  is replaced by  $\phi^{(1)}(t)$ , monotonic and non-decreasing.

Denoting by (\*) the hypothesis of these theorems, we may observe that by virtue of the first theorem of consistency,  $|R, \lambda_n, k| \subset |R, \phi(\lambda_n), k^1|$ ,  $k^1 > k$ , under (\*). Prasad (1954) suggested the possibility of establishing the above theorem by taking less stringent conditions than those in (\*). More generally, he suggested the investigation of problems what he termed 'the unified theorems of consistency'



which assert  $|R, \lambda_n, k| \subset |R, \mu_n, k'|$ , that is, when the type as well as the order both vary.

The formal product of two Dirichlet series  $\sum a_n e^{-\lambda_n s}$  and  $\sum b_n e^{-\mu_n s}$ , arranged according to the ascending order of the sums  $\nu_n = \lambda_p + \mu_q$ , determines the series  $\sum c_n e^{-\nu_n s}$ , where  $c_n = \sum_{\nu_n = \lambda_p + \mu_q} a_p b_q$  and  $\sum c_n$  defines the "Dirichlet Product"

of the series  $\sum a_n$  and  $\sum b_n$  of type  $(\lambda, \mu)$ . Among many theorems for the various kinds of summability of the Dirichlet product  $\sum c_n$ , with reference to summability conditions on  $\sum a_n$  and  $\sum b_n$ , we have the following :

If  $\sum a_n$  is summable  $|R, \lambda, k|$  and  $\sum b_n$  is summable  $|R, \mu, l|$ , then  $\sum c_n$  is summable  $|R, \nu, k+l|$  [Obrechhoff, 1929].

This theorem gives, in particular, that if  $\sum a_n e^{-\lambda_n s}$  is summable  $|R, \lambda, k|$  for  $s = s_1$ , and  $\sum b_n e^{-\mu_n s}$  is absolutely convergent for  $s = s_2$ , where  $\sigma_2 > \sigma_1 \geq 0$ , then  $\sum c_n e^{-\nu_n s}$  is summable  $|R, \nu, k|$  for  $\sigma = \sigma_2$ . In the case when  $k$  is an integer, a direct proof of this result was given by Pati (1960). Again a combination of the theorem given above and a limitation theorem due to Isaacs (1953) gives us that if  $\sum a_n e^{-\lambda_n s_1}$  is summable  $|R, \lambda, k|$  and  $\sum b_n e^{-\mu_n s_2}$  is summable  $|R, \mu, l|$ ,  $\sigma_2 \geq \sigma_1 > 0$ , then

$$e^{-s x} x^{-(k+l)} C_{\nu}^{k+l}(x) \in BV(c, \infty), \sigma > \sigma_2 \geq \sigma_1 > 0.$$

In 1960, Prasad and Pati proved that if  $k$  and  $l$  be integers, then the above result holds also for  $\sigma = \sigma_2 = \sigma_1 > 0$ .

The theory of strong summability of a Dirichlet series is of recent origin and has been worked out mainly by Richert (1955, 1956, 1960), P. Srivastava (1956, 1960) and Prasad and P. Srivastava (1960). This theory in some sense, provides for a unified treatment of both ordinary summability and absolute summability of a Dirichlet series; and in the case of an ordinary Dirichlet series it has been shown to have a more direct relationship with the function-theoretic behaviour of the sum-function and has applications in the asymptotic number theory as well.

If the series  $\sum a_n e^{-\lambda_n s}$  is summable  $[R, \lambda, k, q]$  at a point  $s^*$ , then it is summable  $(k-1) |R, \lambda, k, q|$  for every value of  $s$ , such that  $\sigma > \sigma^*$ . Moreover if, for  $q \geq 1$ ,  $q+1 > 0$ ,  $\sigma^* > 0$ , the above Dirichlet series is summable or bounded  $[R, \lambda, k, q]$ , then, as  $\omega \rightarrow \infty$ ,

$$A_{\lambda}^{k,q}[\omega] = \int |A_{\lambda}^{k-1}(x)|^q dx = \begin{cases} o(e^{q\sigma^*} \omega^{(k-1)q+1}) & \text{(or } O(\dots)), \\ \text{or} \\ o(e^{q\sigma^*} \omega^{kq}) & \text{(or } O(\dots)), \end{cases}$$

according as  $k$  is or is not an integer; and conversely, if for  $q \geq 1$ ,  $k > 0$ ,

$$A_{\lambda}^{k,q}[\omega] = o(e^{q\sigma^*}), \quad \omega \rightarrow \infty,$$

for every  $\sigma > \sigma^*$ , then the given Dirichlet series is summable  $[R, \lambda, k, q]$  for  $\sigma > \sigma^*$ . This result was established by Prasad and Srivastava in 1960 and has many important consequences as mentioned below.

From the above theorem we deduce that there exists an abscissa of summability  $[R, \lambda, k, q]$ , such that, as usual, the Dirichlet series is strongly summable at every point to the right of the line given by this abscissa and is not summable  $[R, \lambda, k, q]$



at any point to the left of this line. With the help of the theorem, values of the abscissae of summabilities  $[R, \lambda, k, q]$  and  $[R, l, k, q]$  have been calculated and it has been shown that the abscissae of these two summabilities are equivalent. It has also been shown that if  $q = 1$ ,  $\gamma_{k,q} = g_k$  and if  $q = \infty$ ,  $\gamma_{k,q} = \sigma_k$  where  $\gamma_{k,q}$  denotes the abscissa of the summability. Considered as a function of  $k$ , this abscissa is convex and considered as a function of  $1/q$  also it is convex.

Prasad and D. P. Gupta in 1965 studied the convergence of ultraspherical lacunary series. They proved the following theorem :

If  $\{n_i\}$ ,  $i = 1, 2, \dots$  be a sequence of positive numbers satisfying the condition  $n_{i+1}/n_i \geq q > 1$ , and the Fourier ultraspherical coefficients  $a_n$  for  $f(x) \in L^2(-1,1)$  in

$f(x) \sim \sum_{n=1}^{\infty} a_n p_n^{\lambda}(x)$  where

$$a_n = \frac{(n+\lambda)\Gamma\lambda\Gamma(n+1)\Gamma(2\lambda)}{\Gamma(\frac{1}{2}+\lambda)\Gamma(\frac{1}{2})\Gamma(n+2\lambda)} \int_{-1}^1 f(x) (1-x^2)^{1-\frac{1}{2}} P_n^{(\lambda)}(x) dx$$

and  $P_n^{(\lambda)}(x)$  is the ultraspherical polynomial of order  $\lambda$  given by the generating function

$$(1-2xz+z^2)^{-\lambda} = \sum_{n=0}^{\infty} z^n P_n^{(\lambda)}(x).$$

be all zero except for  $n = n_i$  then the series  $\sum a_i P_{n_i}^{(\lambda)}(x)$  converges to  $f(x)$  almost everywhere in  $(-1, 1)$  and the series  $\sum_{i=1}^{\infty} (a_i^2/n_i^{2\lambda})$  converges for  $1 > \lambda \geq 1/2$ .

All these papers were written in forties, fifties and sixties and one can discern the stamp of the master on their style.

Among a very large number of references to Prasad's work in mathematical literature mention may be made of those by Hardy and Littlewood (1931), Bosanquet (1933, 1934), Takahashi, Izumi and Kuniyeda (1934), Moursund (1936), Bosanquet and Hyslop (1937), Smith (1938), Hsiang (1946), Cheng and Anderson (1947), Rudin (1955), Zygmund (1935 : in his treatise on 'Trigonometric Series'), Titchmarsh (1938 : in his treatise on 'Introduction to the Theory of Fourier Transforms') and the 'Progress of Science in India during the Past 25 years' (1938 : Indian Science Congress : Silver Jubilee Volume).

### ASSOCIATION WITH SCIENTIFIC BODIES

#### *Indian National Science Academy (formerly National Institute of Sciences of India)*

He was among the earlier elected Fellows of this premier scientific body of the country. He served on its Council, Editorial Board and various committees, and was its Vice-President in 1961-62.

#### *Indian Science Congress Association*

One of the Life Members of the Association, he had the honour of being elected as President of the Section of Mathematics & Statistics (1944-45), General Secretary (Outstation) (1952-55 and 1958-61) and its General President (1965-66).



He was the first mathematician to be elected General President of the Indian Science Congress during its 53 years of existence.

### *National Academy of Sciences of India*

He was elected a Fellow of the Academy even when the number of its Fellows was limited to 30. He had been President of the Physical Sciences Section (1960) its Vice-President and later its President (1965–66).

### *Indian Mathematical Society*

One of the Life Members of the Society, he had the honour of serving on the Council of the Society for many years and of being its President in 1961. During his tenure of office as President, he strengthened its organizational and academic structure.

### *Calcutta Mathematical Society, Bharat Ganita Parishad & Vijnān Parishad*

He was a Vice-President of the Calcutta Mathematical Society and *Vijnān Parishad* and a Member of the Managing Committee of the *Bharat Ganita Parishad*.

### *Foreign Societies*

He was a Member of the London Mathematical Society and *Societè Mathématique de France*, and was a Reviewer for *Mathematical Reviews* of the American Mathematical Society.

## LONG-CHERISHED DREAMS FULFILLED

While in the United Kingdom, on January 1, 1930, B. N. Prasad wrote in his diary about the necessity and desirability of starting an all-India Mathematical Society and of publishing a first-rate mathematical research journal under its auspices. He also wrote about the necessity of starting an Institute of Mathematics in India for the advancement of quality research in the country. Even to him, at that time, these ideas were no more than a dream and it is given to few people in the world that their dreams get fulfilled within their life time. Prasad was one of those fortunate few.

He believed that a Mathematical Society should not only organize annual conferences and publish research journals, but also devise and successfully execute constructive programmes for promoting quality research in the country. With this goal, at his initiative, the Allahabad Mathematical Society was founded in November 1958, with 20 Founder Members and with Dr B. N. Prasad as its first President. The Society decided to publish its new research journal, the *Indian Journal of Mathematics*. During the past 25 years (1958–83) the Society and its *Journal* have stood the tests of time and have earned for themselves an international status and recognition, thus fulfilling his first dream.



As to his second one, in the year 1964, at the initiative of this author, negotiations started between Professor Prasad and Sri Girdharilal Mehta, Chairman of the B. S. Mehta Trust, Calcutta, to set up a Research Institute of Mathematics and Mathematical Physics at Bharwari (Dist. Allahabad)—about 40 kilometers from the city of Allahabad. Professor Prasad had the satisfaction that this new Research Institute, as envisaged by him, will start functioning soon, which it actually did from 19 July 1975, under the sponsorship of the Government of India (through the Department of Atomic Energy), the State Government of Uttar Pradesh, the B. S. Mehta Trust of Calcutta and the B. N. Prasad Memorial Trust at Allahabad (set up by the Allahabad Mathematical Society).

### HIS PRESIDENTIAL AND OTHER ADDRESSES

- 1945 : To the Mathematics & Statistics Section of the Indian Science Congress at Nagpur, he delivered his Presidential Address entitled *The Summability of Fourier series and its conjugate series*, in which he presented an excellent survey of the work done, upto 1944 concerning the summability problems (of various kinds) of a Fourier series and its conjugate series, with a bibliography of 175 research papers at the end.
- 1954 : To the Bundelkhand College (Agra University) at Jhansi, he delivered a Convocation Address telling the new graduates that “*Every organic institution has either to grow up and live or perish. This is all the more true of Centres of Learning. Unless the mind and spirit are progressively brought to higher and yet higher planes of fulfilment, the very purpose of education will be defeated.....How welcome should be any movement that aims at developing a healthy environment around the universities and colleges through social services and academic and cultural activities of the students.*”
- 1960 : To the Physical Sciences Section of the National Academy of Sciences of India at Gorakhpur, he delivered his address on “*The Absolute Summability of Fourier Series*” presenting an up-to-date account of the research work done in this rather recent branch of Summability Theory, his own students having contributed a good deal to it. He himself had a pioneering role in developing this branch.
- 1961 : To the Indian Mathematical Society at Ahmedabad, he delivered a General Address on *Mathematics in India* and a Technical Address on “*Certain aspects of the Researches on Dirichlet Series.*” In the first part he put forward his own ideas and suggestions for improvement of Mathematical Teaching and Research in the country. He pleaded for better service conditions for teachers in colleges and universities, for making our syllabuses up-to-date without making the changes too abrupt and radical, for establishment of centres of research in different parts of the country and for the university authorities cooperating with these centres of research in every possible manner. In the technical part, he presented an account of the work done on the summability of Dirichlet series—ordinary, absolute and strong : Riesz and Abel summability, as also the summability of Dirichlet product of summable series.



- 1963 : To the *Vijnan Parishad* at Delhi he delivered a Presidential Address at a symposium on "Hindi Medium, Mathematical studies and Research (हिन्दी माध्यम, गणितीय अध्ययन और अनुसंधान)," in which he put forward his own ideas about publishing mathematical research papers in Hindi. While on one hand he was of the view that education at the primary and secondary levels must be through regional languages a research journal exclusively in Hindi should not be published till we have seen that Hindi is accepted as an international language.
- 1966 : To the 53rd session of the Indian Science Congress at Chandigarh, he delivered a General Presidential Address. The first part of the address was entitled "*Science in India*" and the second, technical part, was on "*Recent Researches in the Absolute Summability of Infinite Series and their Applications*." In the first part, he suggested a number of steps to be taken at the national level to attract brilliant young men and women to a scientific research career, to be followed while selecting teachers in colleges and universities and while giving them promotions, to be kept in view while permitting young students to go abroad for studies in subjects in which facilities for research exist in India, and for setting up high rank Research Institutes in different parts of the country so that due attention be paid to all aspects of research : pure, applied and development.

In the technical part, which contained a bibliography of over 300 published papers, he presented a detailed and up-to-date account of researches done all over the world in the field of absolute summability of infinite series, with special reference to Fourier series and Dirichlet series. This was the subject to which Professor Prasad devoted his entire professional career.

- 1967 : To the Allahabad Mathematical Society on the occasion of its First Conference at Allahabad a (posthumous) Presidential Address on *Development of Science in India after Independence*, in which he discussed in reasonable detail the developments that had taken place during the first 20 years after gaining independence, in various branches of Science—Physical, Biological, Agricultural etc.

### RECOGNITION BY THE GOVERNMENT OF INDIA

Dr Prasad's efforts towards the advancement of mathematical researches and scientific and mathematical education in the country were repeatedly recognized not only by the learned scientific bodies of India and abroad, but also by the Government of India from time to time. To enumerate some :

- 1951 : In the first International Conference of the Pan-Indian-Ocean Science Congress Association held at Bangalore he was appointed to represent India for the subject of Mathematics.
- 1954 : He was sent as the Scientist Member of Government of India's Five-Member Delegation, under the leadership of the then Vice-President, Dr S. Radhakrishnan, to represent India in the Eighth UNESCO General Conference held at Montevideo (South America). In this trip he also visited a number of universities in the U.S.A. and gave research lectures.



- 1958 : He was sent by the Government of India to represent the Indian Science Congress Association at the 120th Conference of the British Association for Advancement of Science, held at Glasgow. He was also a delegate to represent India at the International Congress of Mathematicians held at Edinburgh and was also invited to deliver lectures at the Cairo University in Egypt.
- 1959 : Appointed by the University Grants Commission, New Delhi, as Member of its Review Committee for Mathematics and Statistics to "*broadly survey and assess the standard of teaching and research and the facilities available for the purpose and to recommend steps to be taken in order to raise the general level of academic attainment and research in Indian universities.*"
- 1960 : Appointed by the Government of India to be a Member of the Committee set up to explore the possibilities of adding some new departments, including that of Mathematics, to the newly-established University of Jabalpur (M.P.). Also appointed a Member of the Governing Council of the Indian Statistical Institute, Calcutta.
- 1963 : In recognition of his meritorious services to the cause of Science and Education in India, the President of India conferred upon him the title of *Padma Bhushan*, the first Indian mathematician to be so honoured.
- 1964 : A year later, he was once again honoured by the President of India by being nominated to be a Member of Parliament (Rajya Sabha). During the only two years he had at his disposal, he made significant contributions to the deliberations of the Rajya Sabha.
- 1965 : As its General President, he was appointed to represent Indian Science Congress Association once again at the Conference of the British Association for Advancement of Science.

### ASSOCIATION WITH INDIAN UNIVERSITIES

He was member of the Court, Executive Council, Academic Council, Faculties of Arts, Science and Commerce and Committee of Courses and Board of Studies in Mathematics of the Allahabad University; of the Executive Council, Court, Senate, Faculties of Arts and Science and Board of Studies in Mathematics of Banaras Hindu University; of the Board of Studies in Mathematics of Agra, Saugor and Baroda universities; and of various such academic bodies of the Patna and Bihar universities. He was also connected with a number of other Indian universities.

### FAMILY LIFE AND SOCIAL VIRTUES

On May 29, 1923, B. N. Prasad was married to Lakshmi Devi, a very accomplished, kind-hearted and cultured lady. They had three children : one son, Prakash Chandra and two daughters, Indu Prabha and Arun Prabha (died 1979), all of whom got well-settled in life. The family lived happily together till the death of Mrs Prasad on September 15, 1954, at Patna after a prolonged illness. This left a big gap in the otherwise happy life of Dr Prasad, who, in order to keep away from feelings of sadness, kept himself extremely busy all the day round.



A strict disciplinarian, Prasad was an ardent devotee of truth and honesty. His straight and upright behaviour, his mathematically precise and exact way of approach towards every problem, his keen desire to see his department flourish in every manner, coupled with his efforts towards achieving this end, and, above all, the nobility of his character, could not but deeply influence any one who desired and had the opportunity to come in close contact with him. A man of highly refined aesthetic taste, polished manners and dynamic and impressive personality, Dr Prasad carried the weight of his learning very lightly. A voracious reader, he hardly allowed to escape any opportunity to learn something new.

Prasad was greatly hospitable at home. He enjoyed entertaining his friends and pupils, particularly when an Indian or foreign scientist or educationist visited Allahabad and stayed with him. The author recalls with pleasure having met some eminent men from all over the world, including Norbert Wiener, Marshall H. Stone, Linus Pauling, P. A. M. Dirac, Mahapandit Rahul Sankrityayan, S. N. Bose, Acharya Narendra Dev, and many others.

B. N. Prasad's was a dedicated life. He passed through many ups and downs. At times he faced the severest of crises, but he kept his banner high in the bivouac of life. It is no exaggeration to say that, to all seekers of truth in Mathematics, he *will live forever among those who have raised the spirit of man that battles against adversity and refuses to be discomfited.*

### HIS LAST DAY

It was the afternoon of Tuesday, January 18, 1966 when after having attended to his daily mail, Dr Prasad was taking tea in his dining room with his younger daughter and a research pupil, Dr T. Singh. The end came suddenly and he collapsed on the table itself even before any medical aid could reach him. Having lived a glorious life, he quit this world at a moment when he had reached the zenith of his glory. May his Soul rest in Peace!

S. R. SINHA

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## NOTES

## BOOKS, ARTICLES AND TALKS

- (i) Prasad wrote several books for school children, thereby bringing to them too the advantages of the ideas and experiences of a seasoned mathematician of the country.

His book on '*Hydrostatics*', meant for the students of the B.A./B.Sc. classes of Indian universities, remained so popular for decades that, in addition to its original English version, it had to be translated into Marathi and Hindi.

- (ii) He wrote a number of articles on topics of scientific and mathematical interest, particularly a few about the lives of famous Indian mathematicians.
- (iii) Through his talks to various student organizations, from time to time, he tried to popularize even a difficult subject like Mathematics.

## ACKNOWLEDGEMENTS

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