# KOLLAGUNTA GOPALAIYER RAMANATHAN

**(13 November 1920 – 10 May 1992)**

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K.G. Ramanathan

# KOLLAGUNTA GOPALAIYER RAMANATHAN

**(1920–1992)**

**Elected Fellow 1965**

KOLLAGUNTA GOPALAIYER RAMANATHAN was a very distinguished mathematician of the front rank well-known in India and abroad for his outstanding achievements in Arithmetic and Analytic Number Theory and in particular, in crucial questions on arithmetical discontinuous groups such as problems of finite generation, commensurability and maximality, Diophantine inequalities involving forms of higher degree including quadratic forms and automorphic functions related to quadratic forms. As a leading mathematician of repute, he played a highly significant role in facilitating the emergence of and further nurturing a post-independence generation of high-calibre mathematicians in India winning for themselves recognition and acclaim and further in generously aiding the general development of mathematical education and research in India. There were many including the present writer privileged to be part of a spell-bound audience during the fifties, sixties and subsequent years listening to his several remarkable courses of lectures on Modern Algebra, Algebraic Number Fields and Function Fields, Modular Functions, Quadratic Forms, Diophantine Approximation, p-adic Analysis etc. watching the writing on the board flowing fast from the piece of chalk in his hand moving ever forward and never having to back-track for any alteration (virtually in the manner of the all-time great mathematician Professor Carl L Siegel whose perfect and legendary lectures were known for admirably timed and absolutely flawless delivery truly worthy of emulation). This is a reasonably quick response to a request from the Editor of INSA Biographical Memoirs of former Fellows of the Academy (even though it is more than fourteen years since Professor Ramanathan passed away on 10 May 1992 in Mumbai).

Professor Ramanathan was born on 13 November 1920 in Hyderabad, Andhra Pradesh as one of the two sons, along with a daughter of Mrs K Gopala Iyer and Mrs. Anantha Lakshmi. Completing his school education from Wesleyan Mission High School, Secunderabad, he studied next in Nizam College, Hyderabad getting his B.A. degree in Mathematics in 1940 and later on his Master's degree from Loyola College, Madras (now called Chennai) in 1942. He worked as Assistant Lecturer in Mathematics at Annamalai University, Chidambaram during 1945-46 and Lecturer in Mathematics at Osmania University for the next two years. Thereafter, while he was a Research Scholar in Mathematics at the University of Madras for some time, he got mathematical advice and direction from Professors R Vaidyanathaswamy and T

Vijayaraghavan in addition to that from a genial and inspiring Rev. Fr. C Racine who was then Professor of Mathematics at Loyola College and a former student of the famous French mathematician Professor É. Artan. The first ten papers of his were written during 1941-47. The year 1948 brought to him the opportunity to go to the Institute for Advanced Study at Princeton and stay there as Lecturer during 1948-51. (It does not seem out of place to express sincere regret for having erroneously mentioned, due to certain reasons which do not need to be gone into now, in an obituary written on Professor Ramanathan in *Acta Arithmetica*, **Vol. LXIV** (1993), 1-6 that in Princeton he was "assistant to Professor Hermann Weyl"). Under the supervision of Professors CL Siegel and Emil Artan, he worked for a Ph D degree from Princeton University. Their profound influence on him in that period was much reflected throughout his mathematical career.

Returning to India from Princeton after securing his doctorate in 1951, he teamed up with Professor K Chandrasekharan at the Tata Institute of Fundamental Research (TIFR), Bombay (now called Mumbai) in 1951. With tremendous dedication, he zealously helped Professor Chandrasekharan in the magnificent mission to create a School of Mathematics at TIFR and work for its steady evolution into an enviable world-class centre for mathematical research as at present. He used all his remarkable expertise and passion for Theory of Numbers to build a number theory school at TIFR. His fine and meticulously delivered lectures along with informal discussions were aimed at budding or matured scholars with an exciting guided tour through the fascinating and glorious mathematical realm of Fermat, Euler, Lagrange, Gauss, Abel, Jacobi, Dirichlet, Kummer, Galois, Eisenstein, Kronecker, Riemann, Dedekind, Minkowski, Siegel, Hilbert, Hecke, Artin, Weil, Chevalley and many other famous mathematicians. With abiding enthusiasm for the propagation of good mathematics and the spread of wholesome mathematical culture, the role played by him was by no means trivial and was instrumental in the moulding and flowering of many a fine mathematician and in the steady improvement of mathematical education and pursuit of research in Mathematics in many of our institutions of higher learning.

The first ten papers written by him prior to his first Princeton visit deal mostly with arithmetical functions of multiplicative type such as divisor functions, Ramanujan's tau function, their congruence properties, Ramanujan's trigonometrical sums and applications and the nature of the product of all the elements in a finite abelian group. Following Siegel's celebrated work on the theory of quadratic forms over the rational numbers and over algebraic number fields and the reduction theory due to P Humbert in the latter case, he studied in two of his papers ([Ramanathan KG (1951)] and [Ramanathan KG (1952)] the properties of the unit groups of quadratic and hermitian forms over number fields, such as their finite generation and the finiteness of their covolume; his paper [Ramanathan KG (1952)] tackles the case of the so-called abelian quadratic forms. In [Ramanathan KG (1959)], he has applied a beautiful general formula due to Siegel concerning lattice points in symmetric bounded convex domains in Euclidean space to obtain a formula for the discriminant of a division algebra which yields, as a nice consequence, the Hasse-Brauer local-global splitting theorem for the case of quaternion algebras over the rational numbers. A complete study of a generalization to number fields of zeta functions associated by Siegel with indefinite quadratic forms is contained in the long paper [Ramanathan KG (1961)].

The next few papers of his contain a systematic study of the equivalence of and representation by quadratic forms over division algebras carrying an involution, properties of the unit groups of these forms such as finite generation and finite covolume and investigation of theta series associated in the manner of Siegel with such quadratic forms, as a prelude to an analytic theory of these forms in this general set-up, in line with Siegel's path breaking work on quadratic forms right from the thirties and continuing well into the fifties (which has now become familiar as the "Siegel Formula" after the appearance of Weil's famous 1964 -65 *Acta Mathematica* papers dealing with this subject). From the foregoing results of his own along with certain methods of Siegel and also some theorems due to Selberg and Borel, one finds in the important paper [Ramanathan KG (1963)] (see also [Ramanathan KG (1957)]) his 'density theorem' and his solution to the problem of constructing infinitely many classes of mutually incommensurable discrete groups of the first kind in classical semi simple groups. Then came his beautiful paper [Ramanathan KG (1963)] settling the question of maximality of discrete subgroups of arithmetically defined classical groups, thereby generalizing earlier results due to Hecke and Maass. The nice paper [Ramanathan KG (1969)] contains as an application of his 'density theorem' a rather surprising converse to Siegel's basic theorem on the unit group of rational quadratic forms f in more than 2 variables being of finite covolume in the orthogonal group O(f) of f Paper [Ramanathan KG (1968)] which establishes the 'dense' nature of the set of values at integral algebraic arguments (belonging to a given algebraic number field K) of 'irrational indefinite' quadratic forms (in $n \geq$ variables) representing 0 non-trivially over K, has the merit that the number of variables needed for the generalisation to go through is just 5 and independent of the degree of K); it was, by the way, meant to be a litmus test for hope that successful generalization (in this special case) to number fields could be a signal to possible validity of the Davenport Conjecture (now referred to as Oppenheim Conjecture), in general, for $n \geq 5$ [as Professor Davenport in his address on 19 November, 1959 as President of the London Mathematical Society had said "...The true condition is probably $n \geq 5$ but I fear that in this subject it is rarely possible to get near to the ultimate truth"(cf. Collected Works of H Davenport, Vol III, 1194)"] and perhaps even for n ? ? . After nearly forty years, Margulis "could get near to the ultimate truth" by studying "the orbit structure of the special orthogonal group SO(f ) acting on $SL(n,\mathbf{R})/SL(n,\mathbf{Z})$" (and indeed reducing all considerations to the case n=3) proved, in 1987, the stunning theorem that the values of every non-degenerate indefinite 'irrational' quadratic form f in $n \geq 3$ variables at integral arguments are dense near 0 and so in R, settling the 'Oppenheim Conjecture'. This was followed by the remarkable papers of Dani and Margulis Invent Math 98(1989), 405-425 as well as of Ratner Ann Math 134(1991), 545-607 confirming the Raghunathan Conjecture about closure of orbits of unipotent flows as well as various other conjectures formulated in this connection, not to forget, the notable paper of Borel and Prasad (Compos Math 83(1992), 347-372) establishing an 'S-arithmetic generalization' of the "Oppenheim Conjecture". Paper [Ramanathan KG and Raghavan S (1970)] is an 'adelic' version of [Ramanathan KG and Raghavan S (1968)] and [Ramanathan KG and Raghavan S (1972)] is a variation dealing with forms of additive type, being an analogue of a result of Davenport and Roth; a subsequent paper on Acta Arith 24(1974), 497-506, the same topic as in [Ramanathan KG and Raghavan S (1972)] clears the 'inflating' impact of the degree of the number field involved. The next paper [Ramanathan KG and Raghavan S (1972)] on automorphic functions is related to the Siegel Formula and 'is a notable contribution to Analytic Number Theory'.

For several years, Professor Ramanathan had been actively interested in the study of published and unpublished work of the great Indian mathematician Srinivasa Ramanujan, expounding, elucidating and extending his beautiful work on singular values of certain modular functions, Rogers-Ramanujan continued fractions and hypergeometric series. Many mathematicians in the West had made a tremendous advance in respect of many aspects of Ramanujan's unpublished work. In the light of the bright prospects for research thus unfolded, he strongly urged many colleagues to take seriously to this fascinating new domain even in the face of 'cold-shouldering' by peers from within, especially since access to Ramanujan's unpublished work had become much easier due to various circumstances. Actually, during the last few months of his life when his right hand was virtually disabled due to Parkinson's disease after retirement from his Senior Professorship at TIFR in December 1985, he continued to forge ahead starting to write a "monograph on continued fractions" dealing with their aspects - "one relating to the hypergeometric series and the other related to basic hypergeometric series". He was heard very often to remark that he was "living on borrowed time". The end came possibly much sooner on 10 May, 1992 at his Mumbai residence, after prolonged serious illness worsened by Parkinson's disease and perhaps by earlier cerebral surgery.

Paper [Ramanathan KG (1980)] contains a complete account of Ramanujan's work on the congruences for 5 and 7 and their powers and for the prime 13. In [Ramanathan KG (1982)] and [Ramanathan KG (1987)], one may find a clever application of Kronecker's limit formula in Analytic Number Theory to establish Ramanujan's astonishing formulae for singular values of Eisenstein series and of elliptic modular invariants. Likewise, in [Ramanathan KG (1984)], the same limit formula is invoked to confirm a large number of Ramanujan's formulae in the 'Lost Notebook' for the evaluation of Ramanujan's continued fraction $R(\tau)$ for certain values of $\tau$ however, in [Ramanathan KG (1984)], many of Ramanujan's evaluations of his continued fraction are shown to follow from two 'fundamental results' attributed to him and also proved. First showing that Ramanujan's results on continued fractions follow simply from three-term relations between hypergeometric series (like in the case of Gauss and Euler continued fractions), [Ramanathan KG (1987)] points out then how their q-analogues lead to many of the continued fractions in the 'Lost Notebook' including a famous one that Andrews and others have considered. In [Ramanathan KG (1990)], one finds proofs for many statements of Ramanujan on modular equations (including 'mixed' ones) within the framework of Ramanujan's results in his Notebook II and also the connection between Ramanujan's results and those of Weber.

Realising the need to establish a good school in Applications of Mathematics, he mooted, in 1975, the idea of a joint TIFR-IISc Programme to function at Bangalore on the campus of the Indian Institute of Science. This has come of age, with the emergence of a viable group of very competent mathematicians specialising in Differential Equations and Numerical Analysis, thanks to his untiring efforts and vision.

Although quite attached to his base in Bombay throughout his 34-year tenure with the TIFR, Professor Ramanathan visited many centres of learning on research and teaching assignments- Institute for Advanced Study, Princeton, Mathematische Institut der Universitaet Goettingen, University of Missouri, St. Louis, University of Alberta, Edmonton etc. He has around 50 research publications to his credit. He was a Fellow of the Indian National Science Academy and of the Indian Academy of Sciences and a Founder Fellow of the Maharashtra Academy of Sciences. He had served as President of the Indian Mathematical Society and as Life President of the Bombay Mathematical Colloquium. As the Editor of the Journal of the Indian Mathematical Society for over ten years, he scrupulously maintained a high standard for the Journal. He was a member of the Editorial Board for *Acta Arithmetica* for nearly three decades. He was a recipient of many national awards- Shanti Swarup Bhatnagar Prize (1965), UGC National Lecturer (1965), Jawaharlal Nehru Fellow(1971-73), Padma Bhushan (1983) and INSA Homi Bhabha Medal (1985).

His interests in English, Telugu and Tamil literature with his unfailing knack for pulling out apt quotations on the spot were just as remarkable as his erudition in music. A good conversationalist, he had been heard to remark once or twice in his later years that the reason for his company being sought was probably that he was considered to be 'well-rounded'! However, his occasional quips could have put off a few too! He shunned publicity as strongly as he avoided those who craved all the time for power and ephemeral glory, say through the media. Those who happened to know him somewhat closely could not have failed to note his simplicity and inner humility. For one whose health was indifferent most of the time, he was generally friendly to those who came to him and generous with help when solicited, unmindful of antecedents, quite in tune with sage Valmiki's aphorism: dosho mahaanatra prapannaanaamarakshane (not to protect refuge-seekers is highly sinful). He was quite opposed to students being 'grabbed' or 'snatched'. The Ph D dissertation of the present writer was written under his supervision. Also Professors K Ramachandra, VC Nanda and Sunder Lal as well as Drs N Sukthankar, Sabita Ghosh and Y Sankaran had worked with him for their Ph D He often used to exhort fellow number-theorists to stick to their own mathematical pursuits diligently without being swayed or overawed by glamour or jargon and truly in the anaasritah karmapalam spirit (not expecting the fruits of action). It had been always his dream that there should be a flourishing number theory school. Summing up, his was an illustrious and colourful personality and he will be greatly missed not only by his wife Mrs. Jayalakshmi Ramanathan and two sons Ananth and Mohan to whom he was deeply attached, but also by his countless friends, admirers and former colleagues.

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