ASHOK PRASAD MAITRA

(05 May 1938 – 11 November 2008)

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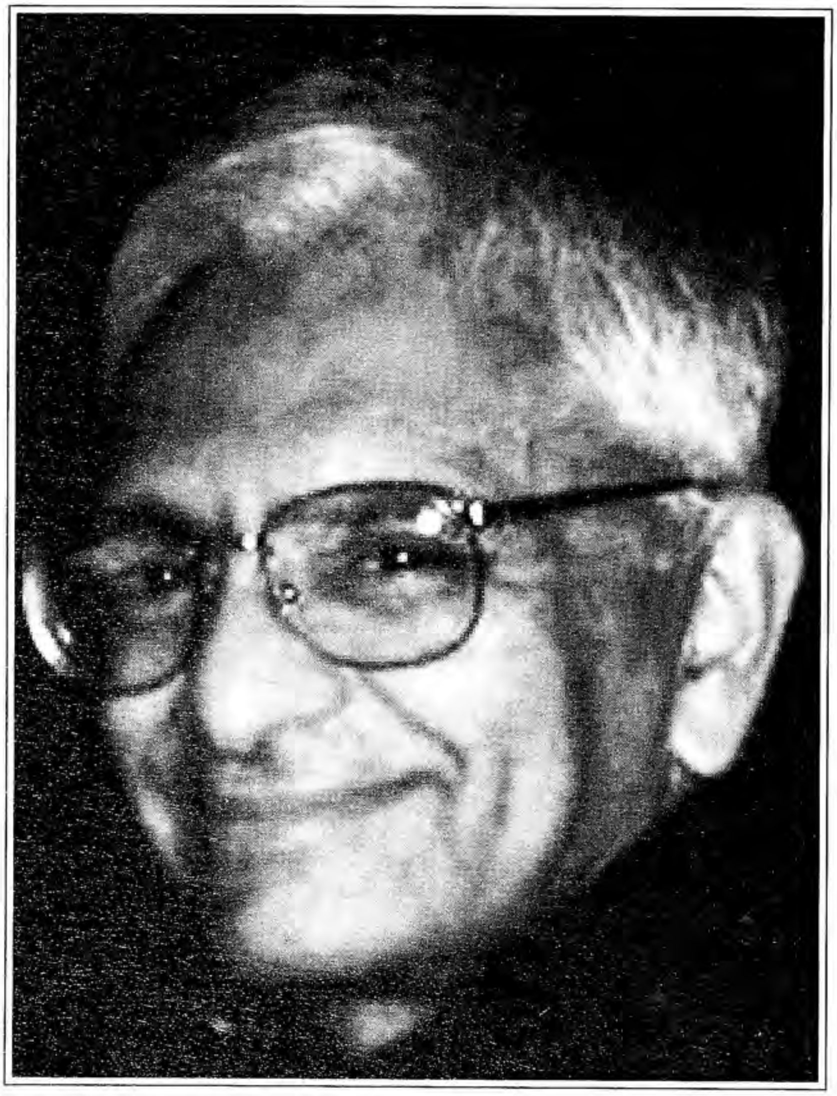

A. Mantha

ASHOK PRASAD MAITRA

(1938 – 2008)

Elected Fellow 1986

ASHOK PRASAD MAITRA, was an outstanding academic with deep and lasting contributions to Descriptive Set Theory, Game Theory, Dynamic Programming, Gambling, Ergodic Theory and Mathematical Statistics. He was the founder of Indian School of Descriptive Set Theory. He belongs to a rare breed of administrators with academic vision and was an inspiring teacher. Above all, he was an ever helpful kind human being. He passed away on November 11, 2008 at a hospital in Minneapolis, USA.

FAMILY BACKGROUND AND EARLY EDUCATION

Ashok Maitra, popularly known as AM was born on May 5, 1938 to Amiya Prasad and Asha Maitra. Amiya Prasad Maitra was a practicing lawyer, who specialized in trademarks and patents. Most of Ashok's childhood was spent in Bombay (since renamed Mumbai), where his father was posted at that time and where he received his early education. He attended Don Bosco High School during 1944-53, from where he passed the Secondary School Certificate Examination in first class. Afterwards, he enrolled in Elphinstone College for the 1953-55 session and completed ISc, securing a first class again. Thereafter, he moved to the Institute of Sciences to pursue undergraduate studies with mathematics as his principal subject and physics as the subsidiary. He was awarded BSc (Honours) degree in 1957 and was placed in the first class with distinction. At both of these institutions, he was recipient of merit scholarships. It was at the Institute of Sciences, he came under the influence of late PR Masani who `introduced him to serious mathematics' and inspired him to take up research and teaching in mathematics as a career. In 1957, he joined the department of statistics of the University of Bombay for the master's programme. He received MSc degree in 1959 and was placed first class first. During 1957-59, he was awarded the Dakshina fellowship of the Bombay University.

PROFESSIONAL CAREER AND ADMINISTRATION

In 1959, AM joined the Department of Statistics, University of California at Berkeley as a graduate student. He worked with David Blackwell on the then developing theory of Dynamic programming for countable state systems and obtained doctoral degree in 1963. David Blackwell had the strongest influence on his research career. While still a graduate student he, along with three colleagues, translated from

Russian the massive and scholarly two volume treatise on Markov processes by EB Dynkin. During 1963-64, he was scientific amanuensis at the Department of Statistics of the Copenhagen University in Denmark. The next academic year, he was a research fellow at the Mathematisch Centrum in Amsterdam, The Netherlands.

Professor Maitra then returned to India to join the faculty of the Indian Statistical Institute (ISI), Calcutta (since renamed Kolkata), on December 20, 1965. During those days there were no clear faculty designations in the Institute. He was made an Associate Professor in 1967 and a Professor in 1972. He came to the ISI at a time when there were a large number of resignations from the faculty. To a great extent, it was he who led the resurgence of the Institute. He, with a few colleagues, did indeed succeed in recovering much of the ground lost by the Institute.

He set an example by his deep commitment to academics. Soon after he joined ISI, he gave a series of lectures on ergodic theory. In 1967, he gave a course of lectures on dynamic programming. Subsequently, the contents of these lectures appeared as a monograph published by ISI. Then followed lectures on descriptive set theory with intense research activity in the subject. Most of the topics covered in the lectures were from the books of Kuratowski, Sierpinski and Lusin as well as journal publications, notably from Fundamenta Mathematicae. With his energy and dynamism, he was able to gather young students around him and build, within a short span, an internationally recognized school of descriptive set theory at ISI. As a discipline, descriptive set theory did not attract serious attention in India until then, except for brief forays by VS Varadarajan in connection with his work in representation theory. AM was to feel later that one of his greatest scientific achievements lay in the fact that he was able to initiate for the first time in India, research in descriptive set theory and to build an Indian school on the subject, ranked at that time with leading centres of research in descriptive set theory in the world.

As a natural follow up of these activities, he was soon lecturing on mathematical logic, basically following the monograph of JR Shoenfield on the subject. With a thorough understanding of the logical foundations of set theory and recursive function theory, a smooth transition was made from classical descriptive set theory to effective set theory, a topic on which he gave a series of lectures. It should be mentioned that all of these activities were in addition to the uninterrupted teaching he undertook, in the regular bachelors and masters courses of the Institute, during this period. A person with diverse interests, during this period he also pursued research in stochastic games, in collaboration with T Parthasarathy. These games were introduced in the early fifties by Shapley. He connected the problem of existence of value for these games to problems in dynamic programming. He had also given a series of lectures on the then developing theory of strategic finitely additive probabilities due to R Purves and W Sudderth. This theory had been initiated some years earlier by L Dubins and LJ Savage.

While at ISI, he held several administrative posts with unsurpassed ability and grace. He was the Director (correct nomenclature being, Member-Secretary, Board of Directors) of the International Statistical Education Center, during 1966-72. This center, a part of the Institute, offers a one year training course in statistics for practicing statisticians from developing countries in Asia and Africa. He served as the Dean of Studies of the Institute during 1975-78. He was the Professor-in-Charge of the Division of Theoretical Statistics and Mathematics for the 1978-80 term. He was officiating Director of the Institute for several months during 1978-79 and again during 1983-84.

He was appointed Director of the Indian Statistical Institute for a five year term in April 1984. He took over at a difficult period when the academic atmosphere in the Institute was being undermined. He spent considerable time and energy to ensure that academic matters remained the priority in the Institute. He took care that students were encouraged and remained in focus in all the programmes of the Institute. It caused immense pain to him to see that certain forces had already become strong and it was getting difficult for him to uphold his ideals and academic vision. An academic to the core, a perfect gentleman and not one to compromise on decency, he felt that he had no choice but to leave the scene. This he did, by resigning from his administrative post on 14th January 1987. He left with grace, dignity and integrity intact. His colleagues were saddened and shocked at the unexpected turn of events. They failed to persuade him to return and continue to lead them. Starting 15th January 1987, he took leave to join the University of Minnesota at Minneapolis. It is here that he started a lasting collaboration with Bill Sudderth on problems in gambling and games. In this work, among other things, he brought in his expertise in descriptive set theory to settle various issues on measurability, regularity of optimal return operators, as well as existence of values in certain games. Subsequently he decided to stay on in the USA and on December 20, 1991, resigned from the services of the Institute. Apart from brief visits away, he stayed and worked at Minneapolis for the rest of his life.

THE PERSON

AM always strived for perfection not only in research but also in everything else he did. It was indeed a treat to hear him, during convocations, announce each student's name with his immaculate pronunciation and perfect phonetics, more so while he was Director of ISEC. He was very caring towards students and enjoyed teaching. Even while he was Director of the ISI, he continued to teach and students had direct access to his office; he would always find time for them in spite of severe pressure of administrative work. Those who attended his courses know how well-organized and lucid his lectures were. Anecdotes, motivations and jokes would flow, like a symphony, throughout his lectures. The side remarks made one realize the subtle points, which are likely to be missed. The excitement of doing mathematics was also conveyed with equal ease, throughout his inspiring lectures. One learnt more than the mathematics by attending his lectures.

He was uncomfortable with official formalities. He never claimed any rank. When he was in the company of students, one could easily mistake him for a senior student. On his way back from office, seeing students at the tea stall next to the gate of the Institute's residential campus, he would join them. For the next one or two hours one would observe a 'strongly mixing' group having serious discussion on all matters, academic as well as non-academic (an apt Bengali word is 'adda'). Mention must also be made of the frequent evening meetings at T Krishnan's place in which AM was the central character and which was joined by several friends and colleagues. AM used to refer to this as the unwinding session. He was absolutely genuine and sincere in whatever he did. It is no wonder, CR Rao, D Basu, G Kallianpur and other seniors had a high regard for him and valued his advice on important matters. He guided eleven research students for doctoral degrees in diverse areas: Ergodic Theory, Descriptive Set Theory, Gambling, Markov Chains and Topology.

He always encouraged students and instilled confidence in them. He shared his ideas freely with them and was averse to being competitive. He was an erudite scholar. One could have a scholarly discussion on almost any topic with him. He had a phenomenal memory; he continuously updated himself on all matters. His favourite topics were sports, politics, literature, history and personalities. He was an enthralling quiz master on various occasions at the Institute hostel functions. He liked watching football. He was an expert player of badminton as well as caroms. After a failed first marriage, Ashok married Indrani in March 1971. Indrani obtained a doctorate from Bose Institute in Calcutta in 1988 for her work in physiology. She is currently a Professor in the Department of Liberal Arts and Sciences of the College of St. Catherine, Minneapolis. Their daughter, Ishani, completed her PhD in philosophy from MIT, in 2002 specializing in women's and gender studies. She is currently an Assistant Professor at Rutgers University and is married to Brian Weatherson, a doctorate in philosophy from Monash University, Australia and an associate professor at Rutgers University.

DOCTORAL STUDENTS

The list contains, year, name, and title of thesis. The first and second in the list were jointly supervised by Ashok Maitra and JK Ghosh.

1968 Subrahmanyan Natarajan *Contributions to Ergodic Theory.*

1970 Bhamidi Visweswara Rao *Studies in Borel Structures.*

1973 Marepalli Bhaskara Rao. *Some Contributions to Measure Theory.*

— Kopparty Padmanabha Surya Bhaskara Rao. *Studies in Boolean Algebras and Measure Theory.*
— Asit Baran Raha *Maximal and Minimal Topologies.*
— Nannimangalam Dorairajan Prabhakar *A Study of Dynamic Programming and Gambling Systems.*
1979 Shashi Mohan Srivastava *Studies in the Theory of Measurable Multifunctions.*
— Haimanti Sarbadhakari *Some Contributions to Descriptive Set Theory*
1980 Subramanian Ramakrishnan *Finitely Additive Markov Chains.*
1983 Velamur Varadhan Srivatsa *Measurable Sets in Product Spaces and their Parametrizations.*
1987 Rana Barua *Studies in Set Theoretic Hierarchies: From Borel Sets to R-Sets.*

Books and Monographs

1965 *Markov Processes Vols. I and II, by EB Dynkin* Translated from Russian, Cotranslators: J. Fabius, V Greenberg. G Majone; Springer-Verlag, Berlin.
1967 *Dynamic Programming in Markov Chains* RTS publication, SM 67-1; iii+107 pp Indian Statistical Institute, Calcutta.
1996 *Discrete Gambling and Stochastic Games* Coauthor: William D Sudderth Applications of Mathematics, Springer-Verlag, New York. xii+244 pp ISBN 0-387-94628-4. [MR 1382657 (99c:60091) 60G40 (28A12 90D15)]

SCIENTIFIC CONTRIBUTIONS

The very first research contribution of AM was to the theory of sufficient statistics, jointly with Edward Barankin. It was a characterization of exponential families among certain class of densities as precisely those for which a sufficient statistic exists. This was a nontrivial generalization, of classical results, to densities corresponding to independent but not necessarily identically distributed random variables. The minimal dimension of the sufficient statistic, when it exists, was also found. The works [4] and [18] are concerned with problems in ergodic theory. In [4], properties of stable transformations were studied and conditions for such a transformation to be mixing were provided. This short paper gave rise to several interesting questions which were taken up later by K Viswanath and S Natarajan. A transformation T on a space $(\Omega; \mathscr{A}; P)$ is stable if for every $A; B \in \mathscr{A}$ the limit $\lim P(T^{-n}A \cap B)$ exists. The transformation is called mixing if this limit equals $P(A)P(B)$. The work [18] on integral representation of invariant measures is an elegant unification of the existing work, besides generalizing it to an abstract set up. The novelty lies in the fact that the problem is treated using sufficiency and regular conditional probabilities. The main theorem in the abstract setting says the following. Let $(X; \mathscr{B})$ be a space on which every probability measure is perfect and let T be a separable family of transformations of X to itself. Then every invariant measure is representable as a unique mixture of ergodic measures. Related to disintegration, but in a different vein, he showed in [16] that if (X_n) is an exchangeable sequence of random variables taking values in a separable metric space, then there is essentially one σ-field which makes the sequence conditionally independent. This result complements an earlier result of Olshen. An entirely different issue regarding factorization was taken up in [27] and [29]. The first of these, jointly with David Blackwell, provides an elementary proof of the following known result. If a separable metric space Y is such that for any Polish space X, any probability on $X \times Y$ admits a factorization on Y (given X), then Y is measurable w.r.t. every probability on the metric completion of Y. The converse also holds. In the second article, jointly with S. Ramakrishnan, relations between perfectness of a probability P on $(Y; \mathscr{C})$ and the existence of finitely additive factorization for any probability on $Y \times R$ with marginal P were obtained.

His contributions to descriptive set theory, gambling and game theory are the most penetrating and deepest. First we shall explain some contributions of AM to descriptive set theory. He investigated existence of selections in a wide range of contexts–closed set valued maps, partitions, sets in product spaces with large sections. Some of these results, while unifying the existing ones, clarified the underlying principles and provided deep insights. Let us recall that a Polish space is a complete separable metric space. A Borel set is a set that can be obtained, by performing countably many times, the operation of complementation and countable union on the collection of open sets. Analytic sets are continuous images of Borel sets, while coanalytic sets are complements of analytic sets in Polish spaces.

The very first work of AM in set theory [8] introduces the concept of Blackwell spaces and studies their properties. In particular, he showed that there are coanalytic sets which are not Blackwell spaces. The interest in this result is due to the fact that every analytic set was known to be Blackwell. This concept leads to mathematically interesting and difficult problems. Some of these problems were later settled by KP S Bhaskara Rao and Rae Shortt.

In the joint work [9] with Ryll-Nardzewski, he showed the following. Let Y be a universal analytic set and let B be an analytic set with B^c having the cardinality of the continuum but not containing a perfect set. Then Y and B are not Borel isomorphic. Since existence of sets like B is consistent (with ZFC), the negation of the statement 'any two analytic non-Borel sets are isomorphic' is consistent. The importance of this result stems from the fact that there is only one uncountable Borel set, up to Borel isomorphism, in Polish spaces. This short article revived interest in the problem and a few years later the efforts of logicians DA Martin, L Harrington and M Steele showed that the above statement is actually equivalent to the determinacy of analytic games.

The joint work [13] with Kuratowski, a far reaching extension of an earlier result of the Bourbaki School, determines the class of Borel selector for partition of Polish spaces into closed sets. Later these results were refined in [17] to obtain the sharpest possible conclusions. The main idea of this later work was to obtain, using a result of Arhangelski, a nice order relation on the Polish space. This technique not only gives a selector, but also provides an algorithm for the selector. These results were later generalized to more complicated partitions by RR Kallman, RD Mauldin and DE Miller. One of the profound discoveries of David Blackwell is the connection between winning strategies in certain games and separation properties of analytic sets. AM had successfully used these game theoretic methods to study several well known properties of analytic sets. In [10], the co-reduction principle for analytic sets was treated. This says the following. Given two analytic sets A and B in a Polish space X, we can find two analytic sets $\tilde{A}$ and $\tilde{B}$ such that $A \subset \tilde{A}$, $B \subset \tilde{B}$, $A \cap B = \tilde{A} \cap \tilde{B}$ and $\tilde{A} \cup \tilde{B} = X$. This result was proved in the setting of general topological spaces, where analogues of analytic sets are known as Souslin sets. In [12], he again uses the game theoretic ideas of Blackwell to produce two coanalytic sets which cannot be separated by a Borel set. In [19] he uses ordinal solutions, discovered by David Blackwell, of certain games to define constituents of coanalytic sets and showed that these constituents possess properties similar to the classical constituents, like the boundedness property.

Suppose, we have a Borel set A in the unit square $[0; 1] \times [0; 1]$ having each x-section non-empty. We wish to place a Borel graph in A. Let A_1 be the set of points $x \in [0; 1]$ such that $(x; y) \in A$ for some $y \in [0; 1/2]$ and A_2 be the set of points x such that $(x; y) \in A$ for some $y \in [1/2; 1]$. Then for points $x \in A_1 \cap A_2$, we have to make up our mind whether to choose a point y from $[0; 1/2]$ or a point y from $[1/2; 1]$ to place $(x; y)$ in the proposed graph. If we have a reduction principle at this stage, then the matter is easily settled for us. The profound observation is that the converse is also true. That is, availability of appropriate reduction principle does indeed solve the issue of placing a Borel graph in A. This was at the root of the discussion in [14], where, equivalence between weak reduction principles and existence of measurable selectors for closed-set valued multifunctions was established. The work [25] unified a large number of, apparently unrelated, selection theorems. This unification led to a remarkable simplification of the theory as well. Jointly with VV Srivatsa, in [28] and [30], these methods were used to obtain parametrization results for Borel sets with large sections. The work [24] contains a neat basis theorem which says that a π^1_1 set, under a simple additional condition, includes a Δ^1_1 point. This theorem, once again, not only unified several known selection results, notably those of John Burgess (which itself was a generalization of certain results of H Sarbadhikari, SM Srivastava and G Debs), PG Hinman, SK Thomason and G Hillard, but also provided effective versions. In [22], an elegant proof is provided for a deep and classical result of Lusin: an analytic set in a product X×Y of Polish spaces, for which each x-section is countable, can be enclosed in a countable number of Borel graphs. The last work of AM in set theory [55], jointly with Rana Barua, is the identification of the additive lightfaced hierarchies in an infinite product of Polish spaces with those of the hierarchies in the same product space but now coordinate spaces having discrete topologies. This is done in the Levy-Solovay model of set theory, making a clever use of the Gandy-Harrington topology. In an obituary note MG Nadkarni, a colleague of AM for many years, acknowledged the influence of AM and the descriptive set theory school in his work, particularly in his book 'Basic Ergodic theory'.

We shall now turn to the contributions of AM to the theory of games, gambling and dynamic programming. For a dynamic programming problem, we have (i) a set I, 'state space'; (ii) a set A, 'action space'; (iii) a map q which associates with every point of $I \times A$ a probability on I, 'law of motion' and (iv) a function from $I \times A$ to R, 'reward function'. There is one player. If he is in state i and chooses action a, the system moves to a state according to the probability $q(.\,|\,i; a)$ and at the same time he receives a reward $r(i; a)$. This is played every day. A plan for the player tells him how to choose actions on each day, knowing the past. Given a plan, the total reward can be defined as cesaro averages of expected daily rewards, or sum of discounted rewards etc. Problems are to understand the optimal reward function (this depends on the initial state), find out optimal or near optimal plans and their nature (Markovian or stationary).

For a stochastic game we have two players and (i) a set S, state space; (ii) sets A and B, action spaces for players I and II respectively; (iii) a map q which associates with every point of $S \times A \times B$ a probability on S, law of motion and (iv) a real valued map on $S \times A \times B$, the reward function. If the state is s and players, knowing all the past, choose actions a and b simultaneously, then the system moves to a state according to the probability $q(.\,|\,s, a, b)$ and at the same time Player I receives reward $r(s; a; b)$ from player II. Strategies are defined analogous to plans described above. This is played every day. Here also there are several ways of defining final reward, which player I tries to maximize and player II tries to minimize. The main problems are the existence of value, its dependence on initial state and optimal or near optimal strategies for the players.

For a gambling problem we have (i) a set F, 'fortune space'; (ii) a map Γ which associates with each $x \in F$, a set $\Gamma(x)$ of finitely additive probabilities on F, 'gambles available at x' and (iii) a real valued function u on F, 'utility function'. A strategy for the gambler is a map σ from the set of finite sequences (including the empty sequence) of fortunes to gambles such that the gamble associated with $(x_1, x_2, \ldots; x_n)$ is in $\Gamma(x_n)$. Every strategy determines, in a canonical way, a strategic probability, denoted again by σ, on the history space F^∞. A plan (σ; t) for the gambler consists of a strategy σ and a stop rule t. The utility of a plan is $\int u(h_{t(h)})d\sigma$. The optimal return function $U(x)$ is the maximum of $u(x)$ and sup $u(\sigma; t)$. Here the sup is over all plans available at x. These are leavable gambling houses, simply because the gambler is allowed to leave as dictated by the stop rule t. One can consider non-leavable houses also, where one must play for ever. In this case $u(\sigma)$, utility of a strategy σ, is defined as $\limsup_t u(\sigma; t)$. The optimal return function in the non-leavable case is supremum of $u(\sigma)$ over all strategies available at x.

AM considered the dynamic programming problem for finite action, countable state space systems and proved, in [2,3], the existence of optimal policies in the discounted case and non-existence in the undiscounted case. In [5,6], he considered Borel state space, compact action space and upper semicontinuous reward function. Measurability of the optimal reward function and existence of stationary policies were shown. One of the main tools used is the Dubins-Savage selection theorem, reflecting perhaps an expertise from his early days. He returned, jointly with Sudderth, to dynamic programming (negative) problems in [39] and provided a transfinite inductive algorithm to obtain the optimal reward operator. The novelty is the use of a deep set theoretic result which he used earlier in the gambling theory context (and appears below). His monograph on dynamic programming gives a neat and systematic exposition of the theory at the time of its publication. It has three chapters, one on discounted case and the other two for the undiscounted case, for finite state spaces and for countable state spaces respectively. A second monograph, dealing with continuous state and action spaces, was envisaged but did not take shape.

The seminal paper [31], jointly with L Dubins, R Purves and W Sudderth, settles a long standing problem in non-leavable gambling theory. If the fortune space is Borel, each gamble is countably additive when restricted to the Borel σ-field, the set of pairs $(x; \gamma)$ where γ is a gamble available at x is an analytic set, and if the utility function u is bounded upper analytic, then the optimal return function V is upper analytic too, in particular, it is universally measurable. In the process, a transfinite inductive algorithm was obtained for the value function. This article also provides a basic link between the finitely additive theory and the classical countably additive theory. The novelty is that for the first time, the concept of inductive definability, due to Moschovakis, was invoked in this arena. This set theoretic result says, loosely speaking, that a monotone operator defined on all subsets which `respects' coanalytic sets has its minimal fixed point to be coanalytic and this minimal fixed point can be obtained from transfinite induction, starting from the empty set.

The article [32], with Sudderth and Purves brings in another beautiful connection with set theory. In the context of Borel measurable leavable gambling problems with utility function not bounded below, the statement `optimal return function is always universally measurable' is equivalent to the statement `continuous images of coanalytic sets are universally measurable'. Jointly with Purves and Sudderth, the regularity of the optimal reward operator was established for the finitely additive case in [35] and for the measurable gambling case in [37]. These regularity results are at the root of the realization that non-measurable strategies are unnecessary in certain situations, for example, when the payoff is bounded. The importance of these results should not be under estimated. In fact, one of the intentions of the Dubins-Savage theory was to understand the role of `measurability' and 'countable additivity' of the classical theory.

Suppose we have a gambling house with its optimal reward operator. What is the largest gambling house (called saturation) which admits the same reward operator. In a different terminology, this was answered by Dellacherie and Meyer by using deep and difficult arguments. In [48], jointly with Sudderth, a neat solution of this problem was presented. This solution involves a careful understanding of the effective analogue of Mokobodzki capacity. In [50], jointly with Dubins and Sudderth, he introduced gambling problems that are invariant under group actions. The interesting point is that dynamic programming problems can be viewed as invariant gambling problems.

His monograph on gambling, jointly with Sudderth, has seven chapters: introduction, gambling houses and the conservation of fairness, leavable gambling problems, non-leavable gambling problems, stationary families of strategies, approximation theorems, and stochastic games. This book provides a much needed update on the subject. Setting aside their set theoretic expertise and avoiding measurability problems, the authors discuss countable fortune space, thus making the book accessible to the beginners too. Indeed, the contents of the book can be described (to use a phrase of John Kelley) as `what every young gambler must know'.

Contributions of AM to game theory began with [7, 11], in collaboration with T Parthasarathy. In the first of these articles, results of Shapley were extended, to the discounted case, for compact state and action spaces, continuous reward function and continuous law of motion. The novelty lies in connecting the game theory problem with an appropriate dynamic programming problem. In the second of these articles, stochastic games with positive reward were considered and existence of value was established. This work, perhaps, was the first to treat general class of games with positive rewards. In [40], jointly with Sudderth, he considered two person zero sum stochastic game on a countable state space with indicator function of a Borel set (in the infinite product of state space) as payoff. Such games are shown to have a value if and only if the Borel set can be approximated `in the sense of value' by open games from outside and closed games from inside. This made perfect sense since open and closed games were already known to have a value. This is very satisfying because it captures a kind of `regularity' even for the existence of value. Since the value function is not a measure, the argument depends on Choquet capacitability theorem. The article [44], jointly with Sudderth, extends the transfinite algorithm provided earlier in gambling set up, to finite action, countable state, two person zero sum stochastic games where the payoff is the limsup of daily rewards. In particular, such games have a value. This result settled a long standing problem in the theory. These results were extended in [41] for stochastic games with Borel state space and compact action sets. Later, these were further extended to arbitrary state and action sets, with finitely additive law of motion and a bounded measurable payoff. In [42], by applying the finitely additive theory of Dubins and Savage, such games were shown to have a value.

Contributions of AM are not restricted to two person games. A fundamental outstanding problem in the theory of many person stochastic games is the existence of Nash equilibria or ε-equilibria. In [54], jointly with Sudderth, the existence of ε-equilibria was established for a class of n person Borel games where the payoff to player i is one or zero according as the process of states remains, forever, in a given Borel set G_i or not. In [56], jointly with Sudderth, he proves the existence of subgame perfect Nash equilibria for n person games when the state space is Borel, action sets are compact, and payoff and law of motion are continuous.

AWARDS AND HONOURS

Awards and honours meant little to Ashok. During his 1972-73 visit to the Department of Statistics at UC, Berkeley, the well known logician, Robert L Vaught requested him to give a series of lectures to his students on classical descriptive set theory which he did. Perhaps, such a recognition brought more satisfaction and a sense of achievement to AM than any award. He was a Fellow of the Indian National Science Academy and elected member of the International Statistical Institute. He was Cullis Memorial Lecturer (1977) of the Calcutta Mathematical Society. He visited, among others, the University of Copenhagen in Denmark, Mathematisch Centrum in Amsterdam, Institute of Information and Automation of the Czechoslovak Academy of Sciences in Prague, Mathematics Institute of Wroclaw University in Wroclaw, Computation Center of the Polish Academy of Sciences in Warszawa, Santa Fe Institute in New Mexico, University of Maryland at Baltimore, University of California at Berkeley, University of Minnesota in Minneapolis and University of Miami in Florida.

LAST DAYS

AM planned a three month visit to the Tata Institute of Fundamental Research towards the end of 2006. He was scheduled to give a series of lectures on game theory. All was set in place by Vivek Borkar. AM was very nostalgic about his early `Bombay Years' and was looking forward to his visit to Bombay with excitement. His students and friends in Kolkata were equally excited that at long last AM was making a trip, of considerable duration, to India. Alas! the invisible hand had other plans.

He suffered a cerebral stroke in August 2006 from which he could not fully recover. He remained bed-ridden, passing through bouts of pneumonia. Though unable to express himself, he could understand when friends called on him and tried to communicate with him. He was mentally alert with memory fully intact! When asked if *A* or *B* was the author of a particular book, he could answer correctly with gestures. Whether he was aware of a particular paper of David Blackwell, he could respond in the affirmative. The extraordinary efforts of Indrani, continuous support and backing from Ishani and Brian kept AM going for two years. It should be mentioned that constant assistance, valuable help and advice extended by the Sudderth family and by the Ramamoorthi family made matters manageable. The sight of a dynamic person being strapped in plastic tubes confined to bed, fully able to understand but unable to express was unbearable for all his friends. Perhaps, it became unbearable for AM too! The end came on November 11, 2008 in Minneapolis.

ACKNOWLEDGEMENTS

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2. 1965 Dynamic programming for countable state systems *Sankhya Ser* **A 27** 241-248 [MR 0200080 (**33** :8220) 90.40] 15

3. 1966 A note on undiscounted dynamic programming *Ann Math Statist* **37** 1042-1044 [MR 0199017 (**33** :7167) 90.40]

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